## Large $P_{\rm T}$ distributions at RHIC and percolation of strings

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**Abstract.** We discuss, in the framework of percolation of strings, the general features of the transverse momentum distributions obtained at RHIC.

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In the framework of the dual string model (DSM) [1,2], in a hadron-hadron or nucleus-nucleus collisions, QCD strings are formed aligned with the collision axis. In the impact parameter plane, these strings look like discs of radius r (the transverse radius of the string), distributed in an interaction area of effective radius R. One can define a percolation transverse density parameter  $\eta$ , such that

$$\eta \equiv \left(\frac{r}{R}\right)^2 N_{\rm s} , \qquad (1)$$

where  $N_{\rm s}$  is the number of produced strings. The variable  $\eta$  is the relevant parameter in percolation theory.

Strings may overlap in the impact parameter plane, forming clusters of N strings. If  $\eta \ll 1$ , strings are isolated and  $\langle N \rangle \simeq 1$ . If  $\eta \gg 1$ , on has overcrowding of strings and  $\langle N \rangle \simeq N_{\rm s}$ . For  $\eta \simeq 1$  we have clusters of all sizes, and sizeable fluctuations,  $\langle N^2 \rangle - \langle N \rangle^2$ . If we define the parameter  $K(\eta)$ , such that

$$1/K \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2} \tag{2}$$

is the normalized cluster size fluctuation, percolation theory says that K goes to infinity in the  $\eta \to 0$  and  $\eta \to \infty$  limits, and has a minimum at some intermediate value of  $\eta$ .

Our main objective with this presentation is to extract from the  $P_{\rm T}$  distributions information on K, and check if the expectations from percolation theory are verified.

In the Schwinger string model for particle production [3] the particle rapidity density dn/dy and  $\langle P_{\rm T}^2 \rangle$  are related by the Gauss theorem, and the  $P_{\rm T}$  distribution is gaussian,  $f(P_{\rm T}) \sim \exp(-P_{\rm T}^2/\bar{P}_1^2)$ , where  $\bar{P}_1^2$  refers to the single string. When clusters are formed a color suppression factor [3,4],  $F(\eta)$ , a decreasing function of  $\eta$ , occurs,

$$F(\eta) \equiv \sqrt{\frac{1 - e^{-\eta}}{\eta}} , \qquad (3)$$

and when one sums over clusters of different sizes [5] the distribution becomes

$$f(P_{\rm T}) \sim \left(1 + \frac{F(\eta)}{K} \frac{P_{\rm T}^2}{\bar{P}_{\rm T}^2}\right)^{-K}$$
, (4)

with  $K(\eta)$  given by (2).

The basic formulae that we need are [5]

$$\mathrm{d}n/\mathrm{d}y = F(\eta)N_{\mathrm{s}}\bar{n}_{1} , \qquad (5)$$

where  $\bar{n}_1$  is the single string density,

$$\langle P_{\rm T}^2 \rangle / \bar{P}_1^2 = \frac{K(\eta)}{K(\eta) - 2} \frac{1}{F(\eta)} ,$$
 (6)

and

$$\frac{\mathrm{d}^2 n}{\mathrm{d}y \mathrm{d}P_{\mathrm{T}}^2} = \frac{\mathrm{d}n}{\mathrm{d}y} \frac{K(\eta) - 1}{K(\eta)} \frac{F(\eta)}{\bar{P}_1^2} \frac{1}{\left(1 + \frac{F(\eta)}{K} \frac{P_{\mathrm{T}}^2}{\bar{P}_1^2}\right)^K} .$$
 (7)

Equation (5) tells us that dn/dy grows slower than  $N_{\rm s}$  (or the number of collisions), (6) tells us that  $\langle P_{\rm T}^2 \rangle$  increases with density, and (7) tells us that at large  $P_{\rm T}$  one moves from a gaussian to a power behaved distribution. Equations (5), (6) and (7), at least qualitatively, are in agreement with experiment.

We shall next relate the radius of interaction R and the average number of strings  $N_{\rm s}$  to the number of participating nucleons,  $N_{\rm part.}$ , and to the energy,  $\sqrt{s}$ . We limit ourselves to the symmetrical situation:  $N_A$  nucleons from one side,  $N_A$  nucleons from the other side, with

$$N_{\text{part.}} = 2N_A \ . \tag{8}$$

For the radius R, conventional nuclear physics suggests that

$$R \simeq R_P N_A^{1/3} , \qquad (9)$$

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**Fig. 1.** Test of relation (12),  $\langle P_T \rangle / \bar{p}_i$  versus  $\frac{1}{N_A^{2/3}} dn/dy$  for  $i = \pi, K, P$  with Phenix 200 GeV data



where  $R_P \simeq 1$  fm is of the order of the proton radius. For  $N_{\rm s}$ , from multiple scattering arguments, we obtain

$$N_{\rm s} \simeq N_{\rm s}^P(\sqrt{s}) N_A^{4/3} , \qquad (10)$$

where  $N_{\rm s}^P(\sqrt{s})$  is the average number of strings in pp collisions at the same energy.

By making use of (9) and (10) in (1), one obtains the energy and number of participants dependence of  $\eta$ :

$$\eta = \left(\frac{r}{R_P}\right)^2 N_{\rm s}^P(\sqrt{s}) N_A^{2/3} . \tag{11}$$

The percolation parameter  $\eta$  increases with the number of strings (or elementary collisions) in pp collisions and with the number of participant nucleons.



Fig. 4. The same for p and  $\bar{p}$ 



Fig. 5. Test of the normalization of (7), dn/dy versus  $N_{\text{part.}}$ 



**Fig. 6.**  $K(\eta)$  extracted from  $\pi^{\pm}$  fits. The behavior expected in percolation is suggested by the data

The single string parameters, particle density  $\bar{n}$ , and average transverse momentum squared,  $\bar{P}_1^2$ , depend on the produced particles, being different for  $\pi, K, p, \ldots$  On the contrary, the quantities  $F(\eta)$ , (3), and  $K(\eta)$ , (2), are universal quantities, related to the distribution of the clusters of strings in the impact parameter plane.

One should note that from (5) and (6), with (11), it follows that there exists a universal relation between  $\langle P_{\rm T}^2 \rangle / \bar{P}_i^2 (i = \pi, K, P)$  and the charged particle density dn/dy in the form

$$\langle P_{\rm T} \rangle / \bar{P}_i = \Phi(\eta) \left( \frac{1}{N_A^{2/3}} \frac{{\rm d}n}{{\rm d}y} \right) ,$$
 (12)

with

$$\Phi(\eta) = \sqrt{\frac{K}{K-2}} \frac{1}{\eta F(\eta)^{3/2}} \frac{1}{\bar{n}_1} \left(\frac{r}{R_p}\right)^2$$
(13)



Fig. 7. Ratios for different distributions  $K/\pi$ ,  $p/\pi$  at two different centralities: (0–5%, dark dots, 60–92%, open dots), in comparison with the data



**Fig. 8.**  $R_CP$  ratios for  $\pi^+, K^+, p$ , as a function of  $p_T$  in comparison with the data

the universality of (12) was tested in a simplified form in [6, 7], and a general test is presented in Fig. 1. The universality is reasonably well satisfied.

In Figs. 2, 3 and 4 we show our plots of the  $P_{\rm T}$  distributions for  $\pi$ , K, p at 200 GeV, in comparison with the Phenix Collaboration results [8].  $F(\eta)$  and  $K(\eta)$  are universal functions,  $F(\eta)$  given by (3) and  $K(\eta)$  to be determined. As we work at fixed energy, the variable  $\eta$  is directly related to  $N_{\rm part.} = 2N_A$ .

The normalization in Figs. 2, 3 and 4 is not free: dn/dy was separately tested in Fig. 5.

The values of the free parameters are  $\bar{P}_{\pi}^2 = (0.25)^2$ ,  $\bar{P}_K^2 = (0.37)^2$  and  $\bar{P}_p^2 = (0.47)^2$ .

In Fig. 6 we show  $K(\eta)$ , as extracted from  $\pi$  data (and used in K and p adjustments). The obtained points are consistent with a function  $K(\eta)$  going to infinity as  $\eta$  goes to infinity, having a minimum at some small value of  $\eta$ , and going to infinity at  $\eta \to 0$ . Points at low  $\eta$ , at present, do not exist.

More detailed comparisons with data are shown in Figs. 7 and 8. In Fig. 7 we show ratios of different distributions for two centralities (0–5% for dark dots and 60–92% for open dots). In Fig. 7 we show the central (0–10%) to peripheral (60–92%) ratios,  $R_{CP}$ , for  $\pi^+$ ,  $K^+$  and p. There is qualitative agreement with the data.

There are however two limitations in our approach. (1) First, we have neglected the factor  $\exp(-m_i^2/\sigma)$  in the Schwinger formula which, with string fusion, becomes  $\exp(-F(\eta)m_i^2/\sigma)$ . This factor makes the ratios  $K/\pi$  and  $p/\pi$  increase with centrality, as seen in experiment. With (5) they are just constant.

(2) Second, with the increase of density the probability of having strings starting from large  $P_{\rm T}$  diquarks increases, adding a new contribution to baryon formation.

These problems are, at the moment, under study.



Fig. 9. Fluctuations (variance/multiplicity) as function of  $N_{\text{part.}}$  from NA49 data, consistent with the vanishing of 1/K at small densities  $(N_{\text{part.}} \rightarrow 2)$  and large densities  $(N_{\text{part.}} \rightarrow \infty)$ . The curve is from [10]

Finally, and coming back to the question of K, as 1/K is nothing but the normalized fluctuation of the number of strings per cluster, see (2), if we believe in fixed  $N_{\rm s}$  percolation theory, with 1/K vanishing at small and large  $\eta$  (or  $N_{\rm part.}$ ), with a maximum somewhere in the middle, we understand the NA49 results [9] on particle production fluctuations [10]. See Fig. 9 for a comparison.

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